LEARNING FROM OBSERVATIONS OF THE MICROWAVE BACKGROUND AT SMALL ANGULAR SCALES

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ABSTRACT

In this paper, we focus our attention on the following question: How well can we recover the power spectrum of the cosmic microwave background from the maps of a given experiment?. Each experiment is described by a a pixelization scale, a beam size, a noise level and a sky coverage. We use accurate numerical simulations of the microwave sky and a cold dark matter model for structure formation in the universe. Angular scales smaller than those of previous simulations are included. The spectrum obtained from the simulated maps is appropriately compared with the theoretical one. Relative deviations between these spectra are estimated. Various contributions to these deviations are analyzed. The method used for spectra comparisons is discussed.

Subject headings: cosmic microwave background—cosmology: theory—large-scale structure of the universe

1. INTRODUCTION

This paper is devoted to the estimation of the cosmic microwave background (CMB) anisotropies from the maps of a given experiment. The goodness of a certain experiment can be tested by using numerically simulated maps similar to those of the chosen experiment. Simulations require a theoretical model for structure formation in the universe. Since we are particularly interested in the analysis of some experiment features, only simulations based on the theoretical model of Sec. 2 are considered. Other models will be studied elsewhere.

From the spectrum of the CMB anisotropies corresponding to a theoretical model –defined by the quantities $C_\ell \equiv \sum_{m=-\ell}^{m=\ell} |a_{\ell m}|^2/(2\ell + 1)$– and the features of a certain experiment, we can build up simulated maps. From these maps, the initial spectrum can be partially recovered. The final spectrum deviates from the initial one. The main goal of this paper is the study of various important contributions to these deviations. The following facts are taken into account: (1) The existence of an angle, θ_{min}, defining a regular network on the sky (pixelization), (2) the use of a Gaussian antenna with a full-width at half-maximum angle θ_{FWHM}. This angle does not fix the pixelization scale. Angles θ_{min} and θ_{FWHM} are independent in spite of the fact that they usually take on similar values; for example, in the case of an ideal detector with $\theta_{FWHM} = 0$, a nonvanishing θ_{min} value could be required by the observational strategy, (3) the fact that only one realization of the CMB sky is available from our position in the universe. This is the cosmic deviation (usually described by the so-called cosmic variance), (4) the partial coverage of the unique available CMB realization. The size –area– and the shape –including possible holes– of the observed regions are important (Scott, Srednicki & White 1994) and, (5) the simultaneous existence of white noise and a partial sky coverage.

The method used in order to recover the angular power spectrum from the simulated maps is described and analyzed along the paper. This method appears to be an interesting alternative to usual methods based on Fourier analysis, which require the use of small maps. See Sáez, Holtmann

& Smoot (1996) and Hobson & magueijo (1996) for interesting applications of Fourier tecniques to the generation and analysis of appropriate patches of the CMB sky. The proposed method is particularly appropriate in the case of maps covering a great region of the sky. These maps can be studied as a whole. They can include holes. Other interesting features of this method are pointed out in Secs. 4, 5 and 6.

The contamination produced by the Milky Way and other galaxies is not studied in this paper. This is not a dramatic restriction for experiments with appropriate frequencies greater than $\sim 80~GHz$ and smaller than $\sim 120~GHz$. For these frequencies, the emissivities from the Milky Way and the extragalactic foregrounds are minimum. At small-intermediate angular scales, temperature fluctuations produced by these emissions are expected to be near 10^{-6}. Observations in the above frequencies are only feasible in the case of satellite experiments. For ground-based and balloon-borne experiments, the atmosphere prevents observations at frequencies lying between $80~GHz$ and $120~GHz$. Observations must be carried out at frequencies much smaller than $80~GHz$ and, then, the contributions of the galaxy and the extragalactic sources must be carefully subtracted in a model dependent way (Tegmark & Efstathiou 1995, Dodelson 1995).

This paper gives a partial answer to the following question: What can we learn from satellite observations ($80~GHz < \nu < 120~GHz$) with high sensitivities and coverages plus accurate simulations of the microwave sky?. Discussing this question, we are contributing to justify the required observations and simulations, which have a high cost in all the senses.

2. THE MODEL

All the simulations presented in this paper correspond to an unique cold dark matter model for large scale structure formation. This model is defined by the following assumptions: (i) after standard recombination and decoupling, no reionizations modified the anisotropies of the CMB,

(ii) the background is flat ($\Omega_0 = 1$), (iii) the cosmological constant vanishes, (iv) scalar fluctuations are Gaussian and their spectrum is scale-invariant, and (v) tensor fluctuations are absent. In this model, the resulting anisotropy depends on the evolved spectrum of the scalar modes, which depends on the primordial spectrum and the quantities involved in the transfer function. This function involves the density parameter of the baryonic matter Ω_B and the reduced Hubble constant h. The values of the parameters h and Ω_B are assumed to be $1/2$ and 0.03, respectively.

The angular power spectrum of the CMB anisotropy is usually normalized by the rms quadrupole produced by scalar modes. In this paper, normalization is based on the estimator Q_{rms-PS}, which is obtained by fitting the observed temperature fluctuations in the case of a scale-invariant primordial density power spectrum. In the absence of tensor modes, experiments measuring at large angular scales –as COBE (Smoot et al. 1992, Bennet et al. 1992, Wright et al. 1992) and TENERIFE (Hancock et al. 1994) lead to estimations of Q_{rms-PS}. In this paper, the C_ℓ coefficients have been taken from Sugiyama (1995) and renormalized according to the four year COBE data ($Q_{rms-PS} \simeq 18 \ \mu K$, Górski et al. 1996).

Let us give some basic definitions and comments, which are used below: The autocorrelation function can be defined as $C(\theta) = C_{\sigma=0}(\theta)$, where

$$C_\sigma(\theta) = \left\langle \left(\frac{\delta T}{T}\right)_\sigma (\vec{n}_1) \left(\frac{\delta T}{T}\right)_\sigma (\vec{n}_2) \right\rangle . \tag{1}$$

The angle between the unit vectors $\vec{n}_1$ and $\vec{n}_2$ is θ. The quantity $(\frac{\delta T}{T})_\sigma(\vec{n})$ is the temperature contrast in the direction $\vec{n}$ after smoothing with a Gaussian beam described by $\sigma = 0.425\theta_{FWHM}$. The angular brackets stand for a mean on many CMB realizations. Function $C_\sigma(\theta)$ can be expanded in the following form:

$$C_\sigma(\theta) = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell + 1) C_\ell P_\ell(\cos\theta) e^{-(\ell+0.5)^2 \sigma^2}. \tag{2}$$

From Eq. (2) one easily obtains the relation:

$$C_\ell(\sigma) = e^{-(\ell+0.5)^2\sigma^2} C_\ell = \frac{32\pi^3}{(2\ell+1)^2} \int_0^\pi C_\sigma(\theta) P_\ell(\cos\theta) sin\theta d\theta . \tag{3}$$

3.   EXPERIMENTS AND SPECTRA

In order to present a systematic study of the uncertainties in the measurement of the CMB spectrum, three auxiliary experiments are considered. The main features of these experiments are now listed:

Experiment (A): $\theta_{min} \neq 0$, $\theta_{FWHM} = 0$, no noise, and partial coverage.

Experiment (B): $\theta_{min} \neq 0$, $\theta_{FWHM} \neq 0$, no noise, and partial coverage.

Experiment (C): $\theta_{min} \neq 0$, $\theta_{FWHM} \neq 0$, uncorrelated noise, and partial coverage.

Experiment (A) corresponds to a perfect detector measuring temperatures at the nodes of a discrete regular grid covering a part of the sky. Some aspects of this experiment can be studied without simulations. The method used in order to extract the spectrum from given maps of the CMB anisotropy is as follows: the autocorrelation function $C(\theta)$ is estimated from Eq. (1) and, then, Eq. (3) is used in order to get C_ℓ quantities. Many pairs of directions $(\vec{n}_1, \vec{n}_2)$ forming a given angle θ are randomly placed on the available maps in order to perform the average involved in Eq. (1). The number of pairs is experimentally fixed (It is verified that a number of independent pairs greater than the chosen one does not lead to a better estimate of the average). An accurate determination of $C(\theta)$ requires various full realizations of the CMB sky. If these realizations are not available, there are errors in the resulting autocorrelation function and, consequently, there are errors in the C_ℓ coefficients given by Eq. (3). Since only a realization of the CMB sky is available, there is an unavoidable indetermination in the CMB spectrum (cosmic uncertainty). Unfortunately, a full coverage of our CMB sky is not available. Up to date, only small regions of the sky have been observed (except in the case of large angular scales). Future satellite experiments could give a more complete coverage, but contaminations due to the Milky way and other galaxies could either require or suggest the rejection of large regions in some maps.

Let us now consider some uncertainties in $C(\theta)$ appearing as a result of the existence of both

an angle θ_{min} separating neighboring nodes of the grid and an angle θ_{max} associated to a partial coverage of the sky. In other words, from Eq. (1), the function $C(\theta)$ can be only obtained in a certain interval $(\theta_{min},\theta_{max})$. For angles smaller than θ_{min}, the map have not any information. As a result of partial coverage, the great number of $(\vec{n}_1, \vec{n}_2)$ pairs required by Eq. (1) is only feasible for angles smaller than a certain θ_{max}. This means that the maps have not information for too small $(\theta < \theta_{min})$ and too large $(\theta > \theta_{max})$ angular scales. This discussion holds for both observation maps and simulated ones. Even if the temperatures have been accurately measured in the grid nodes, the integration involved in Eq. (3) can only be extended to the interval $(\theta_{min},\theta_{max})$ –not to the interval $(0,\pi)$– and, consequently, this integration leads to C_ℓ values different from the theoretical ones. These values define a certain spectrum which is is hereafter called the *modified spectrum* to be distinguished from the *true spectrum* corresponding to the theoretical model under consideration. The true CMB spectrum cannot be directly obtained from Eq. (1) –namely, from the definition of the autocorrelation function– as a result of intrinsic limitations in the maps and, consequently, we must be cautious with any indirect mathematical method creating (modifying) information outside (inside) the interval $(\theta_{min},\theta_{max})$ to recover the true C_ℓ quantities. Further discussion about this point is given in Sec. 6.

The modified spectrum obtained from the maps is not to be compared with the true spectrum but with the *theoretical modified spectrum* obtained as follows: first, Eq. (2) and the C_ℓ coefficients of the assumed model –for $2 \leq \ell \leq 1100$– are used in order to get the function $C(\theta)$ in the interval $(0, \pi)$; afterwards, the values of $C(\theta)$ in the interval $(\theta_{min},\theta_{max})$ and Eq. (3) are used to get the theoretical modified C_ℓ quantities for $40 \leq \ell \leq 1000$.

The theoretical modified spectra corresponding to several values of θ_{min} and θ_{max} are displayed in Fig. 1, where the continuous line corresponds to the true C_ℓ coefficients of the chosen model (see Sec. 2). In the top panel, the value $\theta_{max} = 9°$ is fixed, while the angle θ_{min} takes on the values $2.5'$ (pointed line), $5'$ (dashed line), and $10'$ (pointed-dashed line). From $\ell = 40$ to $\ell = 1000$, the true C_ℓ

quantities can be only recovered with high accuracy for very small values of θ_{min}. In the bottom panel, the value $\theta_{min} = 5'$ is fixed, while the angle θ_{max} takes on the values $4.5°$ (pointed line), $9°$ (dashed line), and $18°$ (pointed-dashed line). From $\ell = 40$ to $\ell \sim 280$, the effect of a varying θ_{max} is significant; however, for $\ell > 280$, this effect becomes very small.

Fig. 1 shows that, for large angles, the theoretical modified spectrum is affected by pixelization ($\theta_{\min}$ value). This fact does not make difficult the evaluation of theories because the theoretical modified spectrum is to be compared with the modified spectrum obtained from the maps, which is affected by pixelization in the same way.

Experiment (B) admits the same discussion as the previous one. Formulae are the same but the beam size does not vanish. Two beams have been considered in this paper: $\sigma = 2.125'$ ($\theta_{FWHM} = 5'$) and $\sigma = 4.25'$ ($\theta_{FWHM} = 10'$).

The $C_\ell(\sigma)$ quantities obtained from simulated maps –corresponding to given values of θ_{min} and σ– must be compared with the theoretical modified quantities corresponding to the same θ_{min} and σ values. The angle θ_{max} must be compatible with the simulation coverage. If the modified spectra are used for comparisons, the uncertainties due to θ_{min} and σ become separated from other uncertainties due to partial coverage, noise and foregrounds. Comparisons with the true $C_\ell(\sigma)$ do not lead to this separation. Other uncertainties in the determination of the CMB spectrum can be analyzed by using simulations (see Sec. 5).

Our estimation of the $C_\ell(\sigma)$ quantities is directly based on the definition of the autocorrelation function (namely, on Eq. (1)). As stated before, pairs $(\vec{n}_1, \vec{n}_2)$ are appropriately located on the maps. This method is so simple and direct that: (1) It applies to the case of any extented map including holes in a natural way and, (2) the analysis of errors in spectra estimates is very simple. Errors seem to be associated to intrinsic limitations of the maps (pixelization, partial coverage et cetera). These limitations lead to problems with the location of pairs $(\vec{n}_1, \vec{n}_2)$ (see Sec. 6).

4. SIMULATIONS

Our numerical simulations are extended to $40° \times 360°$ regions of the sky. These regions are assumed to be uniformly covered and, consequently, the angle θ_{min} –giving the separation between neighboring points- defines the grid of the simulated maps.

Simulations are based on the expansion:

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\ell_{max}} \sum_{m=-\ell}^{m=+\ell} a_{\ell m} Y_{\ell m}(\theta, \phi), \tag{4}$$

with $\ell_{max} = 1100$. The $a_{\ell m}$ coefficients have been generated as statistically independent random numbers with variance $\langle | a_{\ell m} |^2 \rangle = C_\ell e^{(-\ell+0.5)^2 \sigma^2}$ and zero mean. The spherical harmonics have been carefully calculated. These simulations include scales smaller than those considered in previous ones (see Hinshaw, Bennett & Kogut 1995, Kogut, Hinshaw & Bennett 1995, Jungman et al. 1995). The small θ_{min} and σ values considered in our simulations (see below) require the use of large ℓ values giving information about small angular scales.

In a IBM 30-9021 VF, the CPU cost is ~ 11 hours per simulation. The CPU cost for simulations of the full sky has been estimated to be ~ 50 hours.

5. RESULTS FROM SIMULATIONS

We begin with the experiment (B) for several coverages.

Three $40° \times 360°$ simulations (C1 coverage) cover a surface of 43200 square degrees. This is an area slightly greater than that of the full sky (41253 square degrees). The Milky Way mainly contaminates a band of $40° \times 360°$ and, consequently, taking a conservative point of view, we could try to extract the spectrum of the CMB anisotropy by using two simulations (coverage C2), namely, a total area of 28800 square degrees.

In the absence of noise, holes and other errors in the estimation of the spectrum, coverage C1

should lead to a realization of the cosmic uncertainty; in other words, three bands should lead to results comparable to those of a full realization of the CMB sky, at least, for large ℓ values.

The following coverages are also considered: a full band (C3), a band with four $40° \times 40°$ separated squared holes (C4), and an unique $40° \times 40°$ squared region extracted from a band (C5). Results from cases C1 – C5 give interesting information about the uncertainties in the resulting spectrum appearing as a result of the coverage features.

The left and right top panels of Fig. (2) shows the power spectrum obtained from two different C1 realizations. The beam size is $\sigma = 2.125'$ and $\sigma = 4.25'$ in the right and left panels, respectively. In both cases, the grid is defined by the angle $\theta_{min} = 5'$. Solid lines are the theoretical modified spectra corresponding to the chosen values of σ and θ_{min}.

In order to measure the deviations between the theoretical and simulated spectra, the following quantities are calculated and presented in Table 1 : The mean, $M1$, of the quantities $0.69\ell(\ell + 1)C_\ell(\sigma) \times 10^{10}$ (column 3), the mean, $M2$, of the differences between theoretical and simulated values of these quantities (column 4), the mean, MA, of the absolute value of these differences (column 5), and the typical deviation, Σ, of the differences of column 4 (column 6). These quantities are estimated in appropriate ℓ intervals (column 7).

The intermediate and bottom panels of Fig. 2 have the same structure as the corresponding top panels, but in the intermediate (bottom) panel, the dashed line corresponds to the C3 (C5) coverage. From these panels and Table 1, it follows that, as expected, the existence of holes in $40° \times 360°$ bands and, in general, the incompleteness of the sky coverage lead to significant uncertainties in the spectrum.

Simulations have showed that the simulated spectra essentially oscillate around the modified theoretical one. This is a very good news in order to stablish comparisons with theoretical models (see Sec. 6). The presence of oscillations is pointed out by the relation $\mid M1 \mid < M2$, which is

satisfied for every coverage (see Table 1). The fact that $M1$ is always negative –except in the case of the last entry of Table 1– indicates the existence of a systematic error for large ℓ values. The same indication is obtained from the top panels of Fig. 2, where it can be seen that, for large ℓ values, the curves corresponding to simulations lie slightly below the theoretical ones (see Sec. 6 for an interpretation of this fact).

Finally, the experiment (C) has been considered in order to take into account the possible combined effect of uncorrelated noise and partial coverages (for $\ell > 40$). It has been verified that this combined effect is negligible for the coverages C1 - C5 and a noise level of 27.3 μK. Maps involving pure white noise lead to no correlations ($C(\theta) = 0$) for the ensemble, but pure white noise can give nonvanishing C_ℓ coefficients in the case of an unique sky realization (cosmic variance) or in the case of partial coverage. In order to test the importance of the combined effect of white noise and partial coverage, a $40° \times 40°$ map has been built up. This map only involves pure uncorrelated noise at a level of 27.3 μK. The resulting C_ℓ quantities have been extracted as in any other case. The values of $0.69\ell(\ell + 1)C_\ell \times 10^{10}$ have appeared to be smaller than 10^{-2} for any scale. These values are much smaller than those of Figs. 1 and 2 (order unity), which correspond to cosmological signals. In conclusion, the presence of white noise at a level of 27.3 μK can only be important either in the case of coverages much smaller than C5 or in the case $\ell < 40$. According to our expectations, it has been verified that the smaller the coverage, the greater the relevance of uncorrelated noise.

6.   CONCLUSIONS AND DISCUSSION

The use of modified spectra (see Sec. 3) allows us to separate the effects of smoothing and pixelization from other effects. The form of the modified spectra depends on θ_{min}, θ_{max} and σ. These spectra are to be compared with those extracted from observations or simulations. The main problems with the estimate of the modified spectra are now discussed.

In order to obtain $C_\sigma(\theta)$ from a given map, many pairs $(\vec{n}_1, \vec{n}_2)$ are randomly located on the map. Direction $\vec{n}_1$ can be randomly placed on a node of the grid, but then, for a given θ, direction $\vec{n}_2$ does not point towards another node. This mean that the temperature in the direction $\vec{n}_2$ is not known and, consequently, it must be estimated by using interpolations in the grid. This is a mathematical method introducing wrong information. Let us discuss this point in more detail. In order to get true temperatures outside the nodes, we would need a greater resolution in the experiment (or simulation) and greater ℓ values (namely, physical improvements). The fictitious values generated by interpolation can produce an error whose form is not known from theory.

It has been verified that our estimation of the modified spectrum is good for angles lying in the interval $(\theta_{min}, \theta_{max})$, but a small systematic error (see also Sec. 5) seems to appear as a result of the mentioned interpolation. It decreases as θ increases; hence, it is more important for large ℓ values. This error remains small up to $\ell = 1000$ (see Fig. 2). In the case of large θ values, great coverages avoid problems with the location of pairs $(\vec{n}_1, \vec{n}_2)$. According to Sec. 3, full sky coverages could be particularly important in the case $\ell < 280$.

Sky coverage is important. Figure 2 and Table 1 show the deviations between theoretical and simulated spectra for several coverages. As expected, the greater the coverage, the smaller the deviations (Scott, Srednicki & White 1994). Simulations have showed some relevant features of these deviations. It is noticeable that an important part of the deviations shows an oscillatory character around a curve very close to the theoretical one. It can be seen that, even for the $C5$ coverage, the best fitting to the oscillating values is a curve very close to the theoretical one. This fact enhances the interest of moderated coverages as C5 (bottom panels of Fig. 2), which lead to a spectrum very similar to that of C1 coverages (top panels of Fig. 2), at least, for $\ell > 200$ and after removing oscillatory deviations. This is true even in the case of the bottom right panel of Fig. 2, where one of the most oscillating C5 realizations has been showed (realizations of this type are not abundant). More abundant realizations oscillate as in the left bottom panel. These facts –pointed

out by our accurate simulations– enhance the interest of C5 and similar coverages; in particular, if it is taking into account that maps with these features can be obtained by measuring in selected regions with small contamination.

For $\ell > 40$, uncorrelated noise does not appear to be relevant for the coverages considered in this paper; however, this noise could be important in the case of smaller coverages.

If the normalization of the C_ℓ coefficients is performed according to other estimations of Q_{rms-PS} (Smoot et al (1992), Bennet et al (1994), Górski et al (1994), or future estimates), the above C_ℓ quantities would appear either reduced or magnified by the factor $\sim (Q_{rms-PS}/18)^2$; nevertheless, the main conclusions of this paper would remain unaltered because they do not depend on normalization.

References

Bennett et al. 1992, ApJ, L7

Bennett, C.L. et al. 1994, ApJ, 436, 423

Dodelson, S., 1995, submitted to ApJ, astro-ph/9512021

Górski, K.M., et al., 1994, ApJ, 430, L89

Górski, K.M., et al., 1996, astro-ph/9601063, submitted to ApJ. Letters

Hancock, S., et al. 1994, Nature, 367, 333

Hinshaw, G., Bennett, C.L. & Kogut, A., 1995, ApJ, 441, L1

Hobson & magueijo, 1996, astro-ph/9603064

Jungman, G., Kamionkowski, M., Kosowsky, A. & Spergel, D.N., 1995, astro-ph/9512139

Kogut, A., Hinshaw, G. & Bennett, C.L., 1995, ApJ, 441, L5

Sáez, D., Holtmann, E. & Smoot, G. F., 1996, astro-ph/9606164, accepted in ApJ.

Scott, D., Srednicki, M. & White, M., 1994, ApJ, 421, L5

Smoot et al. 1992, ApJ, 396, L1

Sugiyama, N., 1995, ApJ Supplement, 100, 281

Tegmark, M. & Efstathiou, G., 1995, submitted to MNRAS, astro-ph/9507009

Wright, E.L. et al. 1992, ApJ , 396, L13

TABLE 1

COMPARING THEORETICAL AND SIMULATED SPECTRA

θ_{FWHM}	coverage	M1	M2	MA	Σ	ℓ-interval
10′	C1	1.07	-2.77×10^{-2}	4.66×10^{-2}	1.97×10^{-3}	$40 - 1000$
10′	C2	1.07	-2.72×10^{-2}	5.36×10^{-2}	2.21×10^{-3}	$40 - 1000$
10′	C3	1.07	-3.04×10^{-2}	7.85×10^{-2}	3.24×10^{-3}	$40 - 1000$
10′	C4	1.07	-3.69×10^{-2}	1.00×10^{-1}	4.36×10^{-3}	$40 - 1000$
10′	C5	1.07	-3.73×10^{-2}	1.85×10^{-1}	7.2×10^{-3}	$40 - 1000$
5′	C1	1.23	-3.80×10^{-2}	6.91×10^{-2}	2.74×10^{-3}	$40 - 1000$
5′	C2	1.23	-4.06×10^{-2}	7.86×10^{-2}	3.15×10^{-3}	$40 - 1000$
5′	C3	1.23	-4.21×10^{-2}	1.01×10^{-1}	3.98×10^{-3}	$40 - 1000$
5′	C4	1.23	-4.30×10^{-2}	1.25×10^{-1}	5.04×10^{-3}	$40 - 1000$
5′	C5	1.23	-5.44×10^{-2}	2.70×10^{-1}	1.10×10^{-2}	$40 - 1000$
5′	C1	2.20	1.73×10^{-2}	5.49×10^{-2}	3.80×10^{-3}	$40 - 315$

Figure Captions

Fig. 1. Each panel shows the quantity $0.69\ell(\ell+1)C_\ell(\sigma) \times 10^{10}$ as a function of $\log(\ell)$ in various cases. In both panels, the solid line corresponds to $\sigma = 0, \theta_{min} = 0$ and, $\theta_{max} = \pi$ (true C_ℓ quantities). In the top panel: $\theta_{max} = 9°$, $\sigma = 0$, and curves are labelled by θ_{min} in minutes. In the bottom panel: $\theta_{min} = 5'$, $\sigma = 0$, and curves are characterized by θ_{max} in degrees.

Fig. 2. Same as Fig. 1 in other cases. Left (right) panels correspond to $\theta_{FWHM} = 10'$ ($\theta_{FWHM} = 5'$). In all these panels, $\theta_{min} = 5'$ and $\theta_{max} = 9°$. The continuous line shows the modified theoretical spectrum and the dashed line exhibits that extracted from simulations. Top, intermediate, and bottom panels correspond to the C1, C3, and C5 coverages defined in the text, respectively.